HOMEWORK 3 - ANSWERS TO (MOST) PROBLEMS

PEYAM RYAN TABRIZIAN

1. Section 2.3: Calculating limits using the limit laws

2.3.26. 1 (put under a common denominator $t^2 + t = t(t+1)$ and cancel out)

2.3.29. $\frac{1}{2}$ (put under a common denominator and multiply by the conjugate form)

2.3.38. 0 (by squeeze theorem, because $-1 \leq \sin\left(\frac{\pi}{x}\right) \leq 1$)

2.3.47.

(a)(i) 2 (since $|x-1| = x-1$ in this case)
(a)(ii) -2 (since $|x-1| = 1-x$ in this case)
(b) No, since the right-hand-limit and the left-hand-limit are not equal

2.3.58. Let $a = 0$ and $f(x) = \sin\left(\frac{1}{x}\right)$ (or $\frac{1}{x}$), and $g(x) = -f(x)$.

2.3.59. Let $a = 0$ and $f(x) = \sin\left(\frac{1}{x}\right)$ (or $\frac{1}{x}$), and $g(x) = \frac{1}{f(x)}$

2. Section 2.4: The precise definition of a limit

2.4.2. $\delta = 0.7$ (remember, the smaller the δ, the better!)

2.4.4. $\delta = 0.2$ (I picked this because $\left|\sqrt{0.5} - 1\right| \approx 0.28$ and $\left|\sqrt{1.5} - 1\right| \approx 0.22$, and just pick a number slightly smaller than both)

2.4.19. See discussion section! This is an example of the 'easy case' with $\delta = 5\epsilon$

2.4.32. See discussion section! This is an example of the 'complicated case' with $\delta = \min\left\{1, \frac{\epsilon}{19}\right\}$

To get this δ, notice that if $|x-2| < 1$, then $1 < x < 3$, and so $7 < x^2+2x+4 < 19$, so $|x^2 + 2x + 4| < 19$

Date: Wednesday, February 9th, 2011.

2.4.37. This is again an example of the 'complicated case' with $\delta = min\left\{\frac{a}{2}, \epsilon\sqrt{a}\left(1+\frac{1}{\sqrt{2}}\right)\right\}$

To get this δ, notice that if $|x-a| < \frac{a}{2}$, then $\frac{a}{2} < x < \frac{3a}{2}$, and so in particular $\sqrt{x}+\sqrt{a} > \left(1+\frac{1}{\sqrt{2}}\right)\sqrt{a}$ and then:

$$\frac{|x-a|}{\sqrt{x}+\sqrt{a}} < |x-a|\frac{1}{\left(1+\frac{1}{\sqrt{2}}\right)\sqrt{a}} < \epsilon$$

which gives:

$$|x-a| < \epsilon\sqrt{a}\left(1+\frac{1}{\sqrt{2}}\right)$$

The next two are optional, but good for practice:

2.4.42. $\delta = \sqrt[4]{\frac{1}{M}}$

2.4.43. $\delta = e^M$ (where M is negative)

3. Section 2.5: Continuity

2.5.3. -4 (f not defined at -4; neither), -2 (left-hand-side and right-hand-side limits not equal; continuous from the left), 2 (ditto; continuous from the right), 4 (left-hand-side limit does not exist; continuous from the right)

2.5.8. This is my personal opinion, you might disagree with me

(a) Continuous
(b) Discontinuous (because of cliffs and skyscrapers)
(c) Discontinuous (you pay per mile as **whole**, it doesn't matter whether you've traveled 0.9 miles or 0.99 miles)
(d) Continuous

2.5.9. $g(3) = 6$

2.5.27. Continuous because it's a composition of ln (continuous) and a polynomial (continuous), Dom $= (-\infty, -1) \cup (1, \infty)$

2.5.34. $\tan^{-1}\left(\frac{2}{3}\right)$

2.5.40. Yes, you can check that the left-hand-side-limits and the right-hand-side limits are equal! Plug in values for G, M, R if you want to, for example $G = 2$, $M = 5$, $R = 7$

2.5.47. (For extra practice) Define $f(x) = x^4 + x - 3$, then $f(1) = -1 < 0$, $f(2) = 15 > 0$, so by IVT, there is one number c such that $f(c) = 0$.

2.5.56. Use the fact that $\sin(a+h) = \sin(a)\cos(h) + \sin(h)\cos(a)$

2.5.65. Define $f(t)$ to be the altitude of the monk on the first day, $g(t)$ to be the altitude of the monk on the second day, and let $h(t) = f(t) - g(t)$. Then $h(0) > 0$, $h(12) < 0$ (where 0 means $7AM$ and 12 means $12PM$), then by IVT, there is one number c such that $h(c) = 0$, i.e. $f(c) = g(c)$